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To Professor Nico M. Temme  
 Centrum voor Wiskunde en Informatica  
 P.O. Box 94079  
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Dear Professor Temme,

in my research work on the Poisson integral for a ball in non-euclidean spaces I have obtained the following identities for the hypergeometric function:

$$\begin{aligned}
 \sum_{k=0}^{\infty} \frac{\Gamma(\frac{n+k}{2})\Gamma(\frac{k+1}{2})}{\Gamma(\frac{n}{2}+k-1)} t^k \cdot F\left(k, k+n-1; \frac{n}{2}+k; \frac{1-\sqrt{1-t^2}}{2}\right) C_k^{\frac{n-2}{2}}(x) \equiv \\
 \equiv \frac{\Gamma(\frac{n+1}{2})(n-2)}{2\Gamma(\frac{n}{2}+1)} \sqrt{1-t^2} \cdot F\left(n, 1; \frac{n}{2}+1; \frac{1+xt}{2}\right)
 \end{aligned}$$

and

$$\frac{1}{2} + \sum_{k=1}^{\infty} \frac{\binom{2k-1}{k-1} \binom{\frac{n}{2}+k-2}{k-1}}{\binom{\frac{n}{2}+2k-2}{k-1}} F\left(k, \frac{n-1}{2}+k; \frac{n}{2}+2k; x\right) \left(-\frac{x}{4}\right)^k \equiv \frac{\sqrt{1-x}}{2},$$

if  $|x|, |t| < 1$  and  $n \in N$ ,  $n \geq 3$ , where  $C_k^{\frac{n-2}{2}}(x)$  denotes the Gegenbauer polynomial

$$\binom{k+n-3}{k} F\left(-k, k+n-2; \frac{n-1}{2}; \frac{1-x}{2}\right).$$

Since I am not a specialist in the field of special functions, I would like to ask you whether these identities could be directly proven by other techniques.

Yours sincerely,